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Effective field theory of relativistic quantum hall systems

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Abstract: Motivated by the observation of the fractional quantum Hall effect in graphene, we consider the effective field theory of relativistic quantum Hall states. We find that, beside the Chern-Simons term, the effective action also contains a term of topological nature, which couples the electromagnetic field with a topologically conserved current of $2 + 1$ dimensional relativistic fluid. In contrast to the Chern-Simons term, the new term involves the spacetime metric in a nontrivial way. We extract the predictions of the effective theory for linear electromagnetic and gravitational responses. For fractional quantum Hall states at the zeroth Landau level, additional holomorphic constraints allow one to express the results in terms of two dimensionless constants of topological nature.

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1 Introduction

Recently both integer [1, 2] and fractional [3, 4] quantum Hall effects have been observed in graphene. In many respects, graphene behaves like a relativistic system; in particular the integer quantum Hall (IQH) plateaux corresponds to the Hall conductivity (in unit of the quantum (e^2/h) $\nu = 4(n+\frac{1}{2})$, with the factor 4 due to the spin and valley degeneracies and the offset of 1/2 due to the relativistic nature of the low-energy electron spectrum near the Dirac points. Fractional quantum Hall (FQH) states in graphene have also been observed; in particular, many standard fractions in the range $0<\nu<1$ have been seen.

Motivated by the graphene, in this paper we study the relativistic version of the quantum Hall effect. Although electrons in graphene do not behave in a strictly Lorentz invariant fashion (the Coulomb interaction is practically instantaneous), one can still hope to draw physically relevant lessons for graphene when the physics is insensitive to the velocity of propagation of the interaction. We will see that Lorentz invariance imposes stringent constraints on the low-energy behavior of the system, which enable one to express a multitude of physical observables through a small number of parameters. In particular, we find that there are two topological parameters that enter the effective field theory description: the Hall conductivity ν, and a relativistic version of the shift, denoted here as κ. The latter is defined as the offset between the total charge Q and the magnetic flux in units of the flux quantum N_ϕ when the system is put on a sphere with no quasiparticles or quasiholes present: $Q = \nu N_\phi + \kappa$. Together with a single, nonuniversal function of the magnetic field, denoted as $f(b)$, all parity-odd transports of the quantum Hall system are completely determined up to second order in the expansion over momentum. In particular, the system possesses a Hall viscosity [5] proportional to κ. This relationship is the relativistic analog of the exact relationship between the Hall viscosity and the shift in nonrelativistic systems [6].

There is a further simplification for quantum Hall states on the zeroth Landau level (which corresponds to $-2 \le \nu \le 2$ in graphene), in the limit of negligible Landau-level mixing. The holomorphic constraint on the low-energy states determines completely the function $f(b)$ which is otherwise non-universal. In this case, all parity-odd transport is universal to second order in the expansion over momentum. For example, the frequency and momentum dependence of the Hall conductivity is found to be

$$\sigma_{xy}(\omega, q) = \frac{\nu}{2\pi}\left(1 + \frac{2\omega^2}{\omega_c^2}\right) + \frac{\kappa - \nu}{8\pi}(q\ell_B)^2, \tag{1.1}$$

where $\ell_B = \sqrt{\hbar c/eB}$ is the magnetic length and $\omega_c = v_F/\ell_B$, with v_F being the Fermi velocity. Equation (1.1) parallels a similar formula in the nonrelativistic case [7, 8]. The dependence on the shift is exactly the same, after the identification $\kappa \sim \nu\mathcal{S}$, but there is a difference in a constant in front of $(q\ell_B)^2$. This is the reflection of the fact that the electromagnetic current is not a simple lowest-Landau level operator.

2 Power-counting

From now on we set $\hbar = v_F = 1$ and absorb the electron charge e into the magnetic field. The low-energy dynamics in the bulk of a gapped quantum Hall system can be described by a local effective action which is a functional of the external probes. We will turn on both the electromagnetic and gravitational perturbations. We regard the external magnetic field B to be $O(1)$ and consider perturbations of the external gauge field that are of the same order as the background: $F_{\mu\nu} = O(1)$. Denoting the momentum scale by p, one then has $A_\mu = O(p^{-1})$. The perturbations of the metric is assume to be of order one: $g_{\mu\nu} = O(1)$, so, for example, the Riemann tensor is $O(p^2)$.

The term in the effective Lagrangian with the lowest power of p is the Chern-Simons term,

$$S_{\text{CS}} = \frac{\nu}{4\pi}\int d^3x\sqrt{-g}\,\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda, \tag{2.1}$$

and is $O(p^{-1})$. At the next, $O(1)$, order, there is only one gauge invariant scalar $F_{\mu\nu}F^{\mu\nu}$, so the most general contribution to the action at this order is

$$S_\epsilon = -\int d^3x\,\sqrt{-g}\,\epsilon(b), \tag{2.2}$$

where $b = (\frac{1}{2}F_{\mu\nu}F^{\mu\nu})^{1/2}$ and ϵ can be any function of b. We also introduce a unit timelike vector u^μ, $u^2 = -1$, defined through

$$u^\mu = \frac{1}{2b}\epsilon^{\mu\nu\lambda}F_{\nu\lambda}, \tag{2.3}$$

which corresponds to the local frame in which the electric field vanishes. The Bianchi identity $dF = 0$ becomes $\nabla_\mu(bu^\mu) = 0$. The action S_ϵ depends on the metric, and varying the action with respect to the metric one finds that the stress-energy tensor is, to this order $T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu}$, where $P = b\epsilon'(b) - \epsilon(b)$. The stress-energy tensor has the form of that of an ideal fluid.

To construct higher-order terms, it is convenient to use b and u^μ instead of $F_{\mu\nu}$. Both b and u^μ are $O(1)$ in our power-counting scheme. The definition of u^μ involves b in the denominator, but this does not create any problem since the effective theory is supposed to work only at finite magnetic field. The presence of the unit vector u^μ reminds us of the Einstein-aether theory [9], but there are importance difference due to the fact that we are in (2+1) dimensions. One can write down two obvious terms at order $O(p)$,

$$f(b)\epsilon^{\mu\nu\lambda}u_\mu\partial_\nu u_\lambda, \quad f_2(b)u^\mu\partial_\mu b. \tag{2.4}$$

However, by introducing a new function $f_3(b)$ so that $f_2(b) = bf_3'(b)$, the second term can be shown to vanish by integration by parts. There exists, however, one more term at order $O(p)$. The construction of this term involves a topological current which we now describe.

3 Topological current

One notices that the following current is identically conserved,

$$J^\mu = \frac{1}{8\pi}\varepsilon^{\mu\nu\lambda}\varepsilon^{\alpha\beta\gamma}u_\alpha\Big(\nabla_\nu u_\beta\nabla_\lambda u_\gamma - \frac{1}{2}R_{\nu\lambda\beta\gamma}\Big). \tag{3.1}$$

The current has a topological interpretation. The total charge calculated on any space-like surface is simply the Euler character of this surface. The only exception is the special case of a Euclidean space time containing an S^2, where the total charge becomes the Euler character multiplied by the winding number of the $S^2 \to S^2$ map from the 2D spatial slice to the space of unit vector u^μ (this winding number in Lorentz signature is equal to 1). We can motivate this in the simple case when $u^\mu \sim (1, \vec{0})$. Then the charge density associated with the current (3.1) is the scalar curvature of the 2D surface of constant time,

$$J^0 = \frac{1}{8\pi}{}^{(2)}R, \tag{3.2}$$

so the total charge is proportional to the Euler characteristic χ of the hypersurface, $Q = \chi/2 = 1 - g$, where g is the genus of the surface. A more thorough investigation of this new topological current will be presented in [10].

4 The second topological term

We can now add to the effective action a term $\kappa\int d^3x\sqrt{-g}\,A_\mu J^\mu$. Since J^μ is identically conserved, this term is gauge invariant up to a boundary term, similar to the Chern-Simons term. But in contrast to the Chern-Simons term, the new term exists only for background where the magnetic field does not vanish anywhere. This restriction is natural for the effective field theory of a quantum Hall state. Moreover, the term is $O(p)$ in our power counting scheme, thus we have to take that into account when working to this order.

The term under consideration is the relativistic counterpart of the mixed Chern-Simons term $A \wedge d\omega$ which appears in the nonrelativistic case [11, 12]. In the (2+1)D relativistic theory where the spin connection is non-Abelian such a term cannot be directly written down.

Thus, the final action, including all terms to order $O(p)$ is

$$\begin{aligned}\mathcal{L} = &\frac{\nu}{4\pi}\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda - \epsilon(b) + f(b)\varepsilon^{\mu\nu\lambda}u_\mu\partial_\nu u_\lambda \\ &+ \frac{\kappa}{8\pi}\varepsilon^{\mu\nu\lambda}\varepsilon^{\alpha\beta\gamma}A_\mu u_\alpha\Big(\nabla_\nu u_\beta\nabla_\lambda u_\gamma - \frac{1}{2}R_{\nu\lambda\beta\gamma}\Big).\end{aligned} \tag{4.1}$$

It is interesting to note that when $\epsilon(b) \sim b^{3/2}$ and $f(b) \sim b$, the action is fully Weyl invariant. This would be the case if the microscopic theory underlying the quantum Hall state is a conformal field theory.

5 Relativistic shift

The coefficient κ is related to a relativistic version of the shift [11]. The charge density is the variation of the action with respect to A_0. One can see that the total charge on a closed surface comes only from the Chern-Simons and the κ term in the Lagrangian (4.1),

$$Q = \int d^2x\Big(\frac{\nu}{2\pi}F_{12} + \frac{\kappa}{8\pi}J^0\Big). \tag{5.1}$$

By using (3.2) this can be written as $Q = \nu N_\phi + \kappa\chi/2$, where N_ϕ is the total number of magnetic flux quanta threading the manifold and χ is the Euler character of the manifold. This relationship is a relativistic version of the the shift, which is normally defined as $\mathcal{S}$ in the equation $Q = \nu(N_\phi + \mathcal{S})$ on a sphere [11]. We have defined κ to remain finite at $\nu = 0$.

For an integer quantum Hall states with $\nu = N_f(n+\frac{1}{2})$, where N_f is the total number of "flavor" degeneracy of the Landau levels (in graphene $N_f = 4$) the total charge can be found by summing up all charges of the filled Landau levels on a sphere. We find $\kappa = N_f n(n+1)$. Note that $\kappa = 0$ for the $\nu = \pm 2$ states in graphene.

For fractional quantum Hall states in graphene the value of κ is related to the shift $\mathcal{S}$ of the corresponding state in the usual nonrelativistic theory. For illustration let us consider a state with $0 < \nu < 1$ in graphene with complete SU(4) breaking. Among the four zeroth Landau levels, two are completely filled, and a third zeroth Landau level is incompletely filled, and the fourth is completely empty. The lowest Landau level of the Dirac fermion on a sphere with N_ϕ magnetic flux quanta threading it, is identical to the Landau levels of a nonrelativistic fermion on sphere with $N'_\phi = N_\phi - 1$ magnetic flux quanta (in particular it has degeneracy $N_\phi = N'_\phi + 1$). For the latter, the formula $Q = \nu(N'_\phi + \mathcal{S}_{\rm NR})$ is taken as the definition of $\mathcal{S}_{\rm NR}$, which implies $\kappa = \nu(\mathcal{S}_{\rm NR} - 1)$. For example, the $\nu = \frac{1}{3}$ state has $\kappa = \frac{2}{3}$.

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6 Discrete symmetries

Let us assume the the microscopic theory respects C, P, and T, and discuss if the effective theory breaks these symmetries. Recall that [13] under C, $A_\mu \to -A_\mu$; under P, $x^1 \to -x^1$, $A_0 \to A_0$, $A_1 \to -A_1$, and $A_2 \to A_2$; and under T $t \to -t$, $A_0 \to A_0$ and $A_i \to -A_i$. All these symmetries are broken by the background magnetic field, and C is further broken if there is a nonzero chemical potential. But one combination, PT, leaves both the magnetic field and the chemical potential invariant. It is easy to see that all terms we have considered are invariant with respect to PT. When the chemical potential is zero, however, we can classify our terms also with respect to CP and CT. The latter is the particle-hole symmetry of the lowest Landau level. Under these combinations ν, κ, and $f(b)$ all change signs. Therefore the presence of a nonzero ν, κ, or $f(b)$ at zero chemical potential signals a spontaneous breaking of CP and CT symmetries. The $\nu = 0$ IQH state of graphene has $\kappa = 0$, consistent with unbroken CP and CT. On the other hand, it is easy to construct a multiflavor Moore-Read state [14] at $\nu = 0$ which breaks these symmetries.

7 Momentum density

As the first application of the effective field theory, we compute the momentum density T^{0i} in the background of inhomogeneous magnetic field $B = b(x, y)$. To that end, we turn on a perturbation in the g_{0i} component of the metric tensor and read out the momentum density from the action: $\delta S = \int d^3x T^{0i} g_{0i}$. We find

$$T^{0i} = -\epsilon^{ij}\partial_j \left(\frac{\kappa}{8\pi} b + f(b)\right). \tag{7.1}$$

8 FQH states on the zeroth Landau level

We now show that, for the FQH states on the zeroth Landau level with negligible mixing with other Landau levels, the function $f(b)$ is completely determined by the topological coefficients ν and κ. This comes from a holomorphic constraint relating the momentum density and the particle density.

For concreteness, we choose the following representation for the 2×2 Dirac matrices

$$\gamma^0 = \sigma_3, \quad \gamma^1 = i\sigma_2, \quad \gamma^2 = -i\sigma_1, \tag{8.1}$$

for which the free Hamiltonian has the form

$$H = -i\gamma^0\gamma^i D_i - A_0 = -2i \begin{pmatrix} 0 & D \\ \bar{D} & 0 \end{pmatrix} - A_0. \tag{8.2}$$

Here $D \equiv D_z$, $\bar{D} \equiv D_{\bar{z}}$, and we use complex coordinates: $z = x + iy$, $\bar{z} = x - iy$. The $n = 0$ Landau level are $\psi = (\varphi, 0)^T$, where φ satisfies the holomorphic constraint $\bar{D}\varphi = \bar{D}\varphi^* = 0$. Now let us look at the stress-energy tensor,

$$T^{\mu\nu} = -\frac{i}{4}\bar{\psi}\gamma^{(\mu}\overleftrightarrow{D}^{\nu)}\psi, \tag{8.3}$$

assuming a static, but spatially inhomogeneous, magnetic field, and no electric field. For the $0i$ components we can ignore time derivatives as $A_0 = 0$ and the lowest Landau level has zero energy. We see that

$$T^{0i} = -\frac{i}{4}\varphi^* \overleftrightarrow{D}^i \varphi, \tag{8.4}$$

which, by using the holomorphic constraints, can be transformed into

$$T^{0i} = -\frac{1}{4}\epsilon^{ij}\partial_j n, \tag{8.5}$$

where $n = \varphi^*\varphi$ is the particle number density on the lowest Landau level. Comparing that to (7.1), we find that, for FQH states in the zeroth Landau level,

$$f(b) = \frac{1}{8\pi}(\nu - \kappa)b. \tag{8.6}$$

The calculation above neglects possible mixing between Landau levels, as well as the possible corrections to the stress-energy tensor (8.3) due to interactions. Both effects are small when the interaction energy scale is much smaller than the distance between Landau levels $\sqrt{B}$.

9 Response functions

We now compute different response functions of the relativistic quantum Hall states to external fields.

First we compute the Hall viscosity. The Hall viscosity is defined through response to uniform shear metric perturbations. For simplicity we turn on only spatially homogeneous perturbations of the spatial components of the metric, $g_{ij} = \delta_{ij} + h_{ij}(t)$. The relevant term in the action is

$$\kappa \int d^3x\, \sqrt{-g} A_\mu J^\mu = -\frac{\kappa B}{32\pi}\int d^3x\, \epsilon^{jk} h_{ij}\partial_t h_{ik}, \tag{9.1}$$

where we have performed integration by parts. We find

$$\eta_H = \frac{\kappa B}{8\pi}\,. \tag{9.2}$$

The relationship between the Hall viscosity and κ is identical to the nonrelativistic result $\eta_H = n\mathcal{S}/4$ [6] with the substitution $\mathcal{S} \to \mathcal{S}_{\rm NR} - 1$ for grapheme states with $0 < \nu < 1$. Note that the Hall viscosity depends only on the topological number κ. It is natural since η_H can be determined by adiabatic transport and hence should not depend on non-universal functions like $f(b)$.

Next we look at the components of the stress tensor when one turns on a static, spatially inhomogeneous, electric field. The result is

$$T_{ij} = P\delta_{ij} + \frac{\kappa}{8\pi}(\partial_i E_j + \partial_j E_i) - \left(\frac{\kappa}{4\pi} + f'(B)\right)\delta_{ij}\boldsymbol{\nabla}\cdot\mathbf{E}, \tag{9.3}$$

which can be written in terms of the drift velocity $v^i = \epsilon^{ij}E_j/B$ and the shear rate $V_{ij} = \frac{1}{2}(\partial_i v_j + \partial_j v_i - \delta_{ij}\partial\cdot v)$,

$$T_{ij} = P\delta_{ij} - \eta_H(\epsilon_{ik}V_{kj} + \epsilon_{jk}V_{ki}) + \delta_{ij}(\eta_H + Bf'(B))\boldsymbol{\nabla}\times\mathbf{v}. \tag{9.4}$$

The traceless part of the stress tensor reflects the nonzero Hall viscosity of the quantum Hall fluid [5, 15, 16].

The two point functions of currents give us the response to external electromagnetic field, derived from the quadratic part of the effective action in flat space,

$$\mathcal{L} = -\left(\frac{\kappa}{8\pi B} + \frac{f(B)}{B^2}\right)\epsilon^{ij}E_i\partial_t E_j - \frac{f'(B)}{B}E_i\partial_i B. \tag{9.5}$$

In particular, we find the correction to Hall conductivity σ_{xy} (for longitudinal electric fields) at nonzero frequencies and wavenumbers,

$$\sigma_{xy}(\omega, q) = \frac{\nu}{2\pi} + \left(\frac{\kappa}{4\pi} + \frac{2f(B)}{B}\right)\frac{\omega^2}{B} - \frac{f'(B)}{B}\frac{q^2}{B}. \tag{9.6}$$

At lowest Landau level, the formula becomes

$$\sigma_{xy}(\omega, q) = \frac{\nu}{2\pi} + \frac{\nu}{\pi}\frac{\omega^2}{B} + \frac{\kappa - \nu}{8\pi}\frac{q^2}{B}. \tag{9.7}$$

In Galilean invariant systems, the frequency dependence of the conductivity matrix is completely determined, at $q = 0$, by Kohn's theorem [17]. In relativistic systems Kohn's theorem no longer applies. Nevertheless, eq. (9.7) implies that, in Lorentz invariant systems, the ω^2 correction is completely fixed by the filling fraction.

We now discuss the relevance of our formulas for graphene, where the Lorentz invariance of the low-energy free theory is broken by the Coulomb interaction [18]. In the limit of weak Coulomb interaction our formula should work, to leading order of the interaction strength, for integer quantum Hall states. For fractional quantum Hall states in the zeroth Landau level, if one is interested in the response for $\omega \ll q$, the effect of retardation of the interaction should be small. In this case, one can replace the instantaneous Coulomb interaction by a Lorentz invariant interaction without changing the response functions. Thus the q^2 correction to σ_{xy} is reliable for graphene FQH states at the zeroth Landau level.

10 Summary

We constructed an effective field theory description for relativistic quantum Hall liquids. The theory contains one additional topological coefficient besides the Hall conductivity. For states at the zeroth Landau level, there is an additional holomorphic constraint which completely determines the effective Lagrangian in terms of the two topological numbers. Our formalism elucidates the intricate relationship between topology and geometry in the problem. In particular, there is an important term in the Lagrangian with a topologically determined coefficient (κ), but the term itself depends nontrivially on the metric of space.

It would be interesting to extend the theory to higher orders in momentum expansion and compare the predictions for electromagnetic and gravitational responses to the nonrelativistic case [19, 20].

We note that on a manifold with a boundary, conservation of the topological current requires the addition of a boundary action. However, this is only possible if the field u_μ

is parallel to the boundary. If this is not the case, the gauge non-invariance of the action should be absorbed by the anomaly of the boundary theory. The implication of this should be further investigated.

Finally, our effective description may serve as a benchmark for holographic models of quantum Hall effect, many of which have underlying Lorentz invariance (see, e.g., ref. [21]).

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